Vectors and Functions

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The quotient of an *n*-dimensional vector variable or vector-valued function **x** with a scalar variable *t* equals

$$\frac{\mathbf{x}}{t} := \left(\frac{x_1}{t}, \frac{x_2}{t}, \dots, \frac{x_n}{t}\right).$$

The quotient of a scalar variable *x* with an *n*-dimensional vector variable or vector-valued function **t** equals

$$\frac{\Delta x}{\Delta \mathbf{t}} := \left(\frac{x}{t_1}, \frac{x}{t_2}, \dots, \frac{x}{t_n}\right).$$

The quotient of an *n*-dimensional vector variable or vector-valued function **x** with an independent *n*-dimensional vector variable or vector-valued function **t** equals

$$\frac{\mathbf{x}}{\mathbf{t}} := \left(\frac{\mathbf{x}}{t_1}, \frac{\mathbf{x}}{t_2}, \dots, \frac{\mathbf{x}}{t_n}\right) = \begin{pmatrix} \frac{x_1}{t_1} & \cdots & \frac{x_1}{t_n} \\ \vdots & \ddots & \vdots \\ \frac{x_n}{t_1} & \cdots & \frac{x_n}{t_n} \end{pmatrix}.$$

The derivative of an *n*-dimensional vector function **x** with respect to an independent scalar variable *t* equals

$$\frac{\mathbf{dx}(t)}{dt} \coloneqq \left(\frac{dx_1}{dt}, \frac{dx_2}{dt}, \dots, \frac{dx_n}{dt}\right).$$

The directional derivative of a scalar function *x* with respect to an independent *n*-dimensional vector variable **t** equals

$$\frac{dx(\mathbf{t})}{\mathbf{dt}} := \left(\frac{\partial x}{\partial t_1}, \frac{\partial x}{\partial t_2}, \dots, \frac{\partial x}{\partial t_n}\right),$$

The directional derivative of an *n*-dimensional vector-valued function **x** with respect to an *n*-dimensional vector variable **t** equals

$$\frac{\partial \mathbf{x}(\mathbf{t})}{\partial \mathbf{t}} := \left(\frac{\partial \mathbf{x}}{\partial t_1}, \frac{\partial \mathbf{x}}{\partial t_2}, \dots, \frac{\partial \mathbf{x}}{\partial t_n}\right) = \begin{pmatrix} \frac{\partial x_1}{\partial t_1} & \cdots & \frac{\partial x_1}{\partial t_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial x_n}{\partial t_1} & \cdots & \frac{\partial x_n}{\partial t_n} \end{pmatrix},$$

The matrices are square Jacobian matrices, whose $(i, j)^{\text{th}}$ entry is

$$\mathbf{J}_{ij} = \frac{\partial x_i}{\partial t_j} \; or \; \frac{\partial t_i}{\partial x_j}.$$

The inverse function theorem states that the matrix inverse of the Jacobian matrix of an invertible function is the Jacobian matrix of the inverse function.