Circular/Harmonic Motion

Angular speed (velocity) and angular pace (legerity)

- Speed, $v = \Delta s/\Delta t$, Pace, $u = \Delta t/\Delta s$ so $u = 1/v$ and $v = 1/u$ except if u or v are zero
    - Zero speed: no motion but time changes because time is independent
    - Zero pace: no motion but length changes because length is independent

Circular motion in space and time

distance, x; distime, t; radius r (or a or R or A); period radius q; circumference $S = 2\pi r$ = wavelength λ; period $T = 2\pi q$ = wavetime μ; angular velocity, v; angular legerity, u; arc length, s; arc time, w

- Circle in space
    - space angle θ, arc length s, radius r
    - angle in space: $\theta \equiv s/r$ $\quad r = s/\theta,\ s = r\theta$ $\quad 1/\theta = r/s;\ 1/r = \theta/s$
    - angular time rate: $\omega \equiv \theta/t$ $\quad t = \theta/\omega,\ \theta = \omega t$ $\quad 1/\omega = t/\theta;\ 1/t = \omega/\theta$
- Circle in time
    - time angle φ, arc time w, period radius q
    - angle in time, $\varphi \equiv w/q$ $\quad q = w/\varphi,\ w = q\varphi$ $\quad 1/\varphi = q/w;\ 1/q = \varphi/w$
    - angular space rate: $\psi \equiv \varphi/x$ $\quad x = \varphi/\psi,\ \varphi = \psi x$ $\quad 1/\psi = x/\varphi;\ 1/x = \psi/\varphi$

Angular time rates

- Independent variable is time, dependent variable is length
- Angular velocity: time rate of rotation, $\omega \equiv \theta/t$ $\quad t = \theta/\omega;\ \theta = \omega t$
- Wave speed: wavelength per unit time, $v = s/t = S/T = r/q = \omega r$
    - frequency, $f \equiv 1/T = v/S = v/\lambda = v/s = vh = h/u$ $\quad$ wavelength, $\lambda = v/f$
- Wave speed normalized
    - revolutions: If $S = 1$, then $v = 1/T = f$
    - space radians: If $r = 1$, then $s = \theta = \varphi$ and $v = \theta/t = \varphi/t = s/t = \omega = 1/q = 2\pi/T = 2\pi f$
        - $\omega = 2\pi f = 2\pi/T = \theta/t = \varphi/t$ $\quad q = T/2\pi$ $\quad f = \omega/2\pi$

Angular space rates

- Independent variable is length, dependent variable is time
- Angular legerity: space rate of rotation, $\psi \equiv \varphi/x$ $\quad x = \varphi/\psi;\ \varphi = \psi x$
- Wave pace: wavetime per unit length, $u = w/s = T/S = q/r = \psi q$
    - periodicity, $h \equiv 1/S = u/T = u/\mu = s/v = uf = f/v$ $\quad$ wavetime, $\mu = u/h$
- Wave pace normalized
    - revolutions: If $T = 1$, then $u = 1/S = h$
    - time radians: If $q = 1$, then $w = \varphi = \theta$ and $u = \varphi/x = \theta/x = w/s = \psi = 1/r = 2\pi/S = 2\pi h$
        - $\psi = 2\pi h = 2\pi/S = \theta/x = \varphi/x$ $\quad r = S/2\pi$ $\quad h = \psi/2\pi$

Parametric equations

$|\mathbf{x}| = x = \sqrt{(x_1^2 + x_2^2)}$ $\quad |\mathbf{t}| = t = \sqrt{(t_1^2 + t_2^2)}$

$\mathbf{x}(\theta) = r\cos(\theta)\,\mathbf{i} + r\sin(\theta)\,\mathbf{j}$

$\mathbf{x}(t) = r\cos(\omega t)\,\mathbf{i} + r\sin(\omega t)\,\mathbf{j}$ $\quad v(t) = \omega r$ $\quad \mathbf{a}(t) = -\omega^2\mathbf{x}(t)$

$\mathbf{x}(s) = r\cos(s/r)\,\mathbf{i} + r\sin(s/r)\,\mathbf{j}$

$\mathbf{t}(\varphi) = q\cos(\varphi)\,\mathbf{i} + q\sin(\varphi)\,\mathbf{j}$

$\mathbf{t}(x) = q\cos(\psi x)\,\mathbf{i} + q\sin(\psi x)\,\mathbf{j}$ $\quad u(x) = \psi q$ $\quad \mathbf{b}(x) = -\omega^2\mathbf{t}(x)$

$\mathbf{t}(w) = q\cos(w/q)\,\mathbf{i} + q\sin(w/q)\,\mathbf{j}$

Spiral/Helical Motion

The helices are the curves the tangents of which form a constant angle with respect to a fixed plane or a fixed direction. A helix is the geodesic of a cylinder; if we develop the cylinder on which the helix is traced, the helix becomes a straight line. Radius r (or a or R or A); velocity v, arc length s, arc time, w, pitch length P; pitch time, M; pitch angle α; pitch time angle β

Constants

$v = |\mathbf{v}| = \sqrt{(r^2 + b^2)}$ $\quad s = t\sqrt{(r^2 + b^2)}$ $\quad$ pitch length, $P = 2\pi b$ $\quad$ slope, $P/S = b/r$

$u = |\mathbf{u}| = \sqrt{(q^2 + c^2)}$ $\quad w = x\sqrt{(q^2 + c^2)}$ $\quad$ pitch time, $M = 2\pi c$ $\quad$ time slope, $M/T = c/q$

Arc length of one winding

$L = \sqrt{(P^2 + S^2)}$

$\alpha = \operatorname{atan}(P/S) = \operatorname{atan}(b/r)$

$\beta = \operatorname{atan}(M/T) = \operatorname{atan}(c/q)$

Parametric equations

$|\mathbf{x}| = x = \sqrt{(x_1^2 + x_2^2 + x_3^2)}$ $\quad |\mathbf{t}| = t = \sqrt{(t_1^2 + t_2^2 + t_3^2)}$

$\mathbf{x}(\theta) = r\cos(\theta)\,\mathbf{i} + r\sin(\theta)\,\mathbf{j} + (b\theta/\omega)\,\mathbf{k}$

$\mathbf{x}(t) = r\cos(\omega t)\,\mathbf{i} + r\sin(\omega t)\,\mathbf{j} + bt\,\mathbf{k}$

$\mathbf{x}(s) = r\cos(s/r)\,\mathbf{i} + r\sin(s/r)\,\mathbf{j} + (bs/r)\,\mathbf{k}$

$\mathbf{t}(\varphi) = q\cos(\varphi)\,\mathbf{i} + q\sin(\varphi)\,\mathbf{j} + (c\varphi/\psi)\,\mathbf{k}$

$\mathbf{t}(x) = q\cos(\psi x)\,\mathbf{i} + q\sin(\psi x)\,\mathbf{j} + cx\,\mathbf{k}$

$\mathbf{t}(w) = q\cos(w/q)\,\mathbf{i} + q\sin(w/q)\,\mathbf{j} + (cw/q)\,\mathbf{k}$

$\mathrm{d}\mathbf{x}(t)/\mathrm{d}t = \mathbf{v}(t) = -\,\omega r\sin(\omega t)\,\mathbf{i} + \omega r\cos(\omega t)\,\mathbf{j} + b\mathbf{k}$

$\mathrm{d}\mathbf{v}(t)/\mathrm{d}t = \mathbf{a}(t) = -\,\omega^2 r\cos(\omega t)\,\mathbf{i} + \omega^2 r\sin(\omega t)\,\mathbf{j}$

$\mathrm{d}\mathbf{t}(x)/\mathrm{d}x = \mathbf{u}(x) = -\,\psi q\sin(\psi x)\,\mathbf{i} + \psi q\cos(\psi x)\,\mathbf{j} + c\mathbf{k}$

$\mathrm{d}\mathbf{u}(x)/\mathrm{d}x = \mathbf{b}(x) = -\,\psi^2 q\cos(\psi x)\,\mathbf{i} + \psi^2 q\sin(\psi x)\,\mathbf{j}$