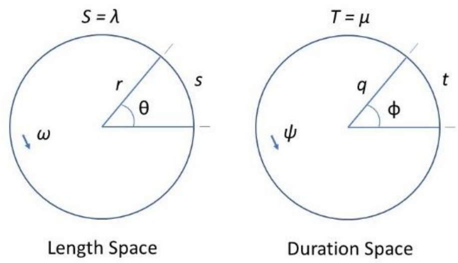
Circular/harmonic motion | revised 2023-04-14

Angular velocity and angular lenticity (wavenumber)

- Velocity, $v = s/\Delta t$, lenticity, $u = t/\Delta s$ so $u = 1/v$ and $v = 1/u$ except if u or v are zero
  - Zero velocity: no motion but time increases because time is independent
  - Zero lenticity: no motion but distance increases because distance is independent
  - Circular motion in length space and duration space:



circumference $S = 2\pi r$ = wavelength λ; period $T = 2\pi q = 1/f$ = wave duration μ; arc length, s; arc duration, t; radius r; period radius q; time rate of rotation ω; distance rate of rotation ψ; angular wavenumber or repetency k; phase|wave velocity, $v_p = \lambda/T = \lambda f = \omega/k$; phase|wave lenticity, $u_p = T/\lambda = k/\omega$

Circular motion in length space with time (3+1)
- length space angle θ, arc length s, radius r
- length space angle $\theta \equiv s/r$

Time rates of rotation

- Independent variable is time, t; dependent variable is arc length, s
- Angular velocity: time rate of rotation, $\omega \equiv \theta/t$
- Wave (phase) velocity: wavelength per unit time, $v = s/t = S/T = r/q = \omega r$
  - frequency, $f \equiv 1/T = v/S = v/\lambda = v/s$
  - wavelength, $\lambda = v/f = S$
  - angular wavenumber or repetency, $k = 2\pi/\lambda$
- Wave velocity normalized
  - revolutions: If $S = 1$, then $v = 1/T = f$
  - length radians: If $r = 1$, then $s = \theta$ and $v = \theta/t = \phi/t = s/t = \omega = 2\pi/T = 2\pi f$
    - $\omega = 2\pi f = 2\pi/T = \theta/t$

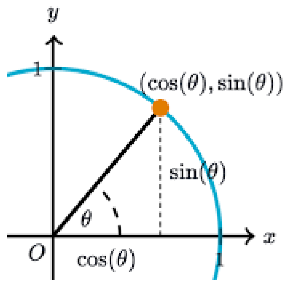
Circular motion in duration space with distance (1+3)
- duration space angle ϕ, arc duration t, period radius q
- duration space angle $\phi \equiv t/q$

Distance rates of rotation

- Independent variable is distance, s; dependent variable is (dis)time, t
- Wavenumber (angular lenticity): distance rate of rotation: $\psi \equiv \phi/s$
- Wave (phase) lenticity: wave duration per unit distance, $u = t/s = T/S = q/r = \psi q$
  periodicity, $h \equiv 1/S = u/T = u/\mu = s/v$
  - wave duration, $\mu = u/h = T$
  - wave duration number, $\ell = 2\pi/\mu$
- Wave lenticity normalized
  - revolutions: If $T = 1$, then $u = 1/S = h$
  - duration radians: If $q = 1$, then $t = \phi = \theta$ and $u = \phi/s = \theta/s = t/s = \psi = 2\pi/S = 2\pi h$
    - $\psi = 2\pi h = 2\pi/S = \theta/s = \phi/s$

Coordinates

length space (r, θ) duration space (q, ϕ)



Wave function for longitudinal wave: cosine wave

length amplitude, A; duration amplitude, B; length phase, $\theta = \psi x$; duration phase, $\phi = \omega t$; distance, s; (dis)time, t; circumference S = wavelength λ; period T = wave duration μ; wave (phase) velocity, v; wave (phase) lenticity, u

length space with scalar time (3+1):

$x = A \cos(\omega t + \theta)$ $a = -\omega^2 x$ in SHM

$y(x = 0, t) = A \cos(\omega t) = A \cos(2\pi f t)$
$y(x, t) = A \cos[\omega(t - x/v)] = A \cos[2\pi f\,(x/v - t)]$ = sinusoidal wave moving in the $+x$ length direction
$y(x, t) = A \cos[2\pi\,(x/\lambda - t/T)] = A \cos(kx - \omega t)$

$\partial^2 y(x, t)/\partial x^2 = (1/v^2)\, \partial^2 y(x, t)/\partial t^2$ length space wave equation

duration space with distance (1+3):

$z = B \cos(\psi s + \phi)$ $b = -\psi^2 z$ in SHM

$\eta(z = 0, s) = B \cos(\psi s) = B \cos(2\pi h s)$
$\eta(z, s) = B \cos[\psi(s - z/u)] = B \cos[2\pi h\,(z/u - s)]$ = sinusoidal wave moving in the $+z$ duration direction
$\eta(z, s) = B \cos[2\pi\,(z/\mu - s/S)] = B \cos(\ell z - \psi s)$

$\partial^2 \eta(z, s)/\partial z^2 = (1/u^2)\, \partial^2 \eta(z, s)/\partial s^2$ duration space wave equation

Wave function for transverse wave: sine wave
…

Parametric equations

$|\mathbf{r}| = r = \sqrt{(r_1^2 + r_2^2)}$ $|\mathbf{q}| = q = \sqrt{(q_1^2 + q_2^2)}$

$\mathbf{r}(\theta) = r \cos(\theta)\, \mathbf{i} + r \sin(\theta)\, \mathbf{j}$
$\mathbf{r}(t) = r \cos(\omega t)\, \mathbf{i} + r \sin(\omega t)\, \mathbf{j}$; $v(t) = \omega r$; $\mathbf{a}(t) = -\omega^2 \mathbf{x}(t)$
$\mathbf{r}(x) = r \cos(x/r)\, \mathbf{i} + r \sin(x/r)\, \mathbf{j}$

$\mathbf{q}(\phi) = q \cos(\phi)\, \mathbf{i} + q \sin(\phi)\, \mathbf{j}$
$\mathbf{q}(s) = q \cos(ks)\, \mathbf{i} + q \sin(\psi s)\, \mathbf{j}$; $u(s) = \psi q$; $\mathbf{b}(s) = -\psi^2 \mathbf{t}(s)$
$\mathbf{q}(z) = q \cos(z/q)\, \mathbf{i} + q \sin(z/q)\, \mathbf{j}$