Circular/harmonic motion | revised 2020-02-08

Angular velocity and angular lenticity (wavenumber)

- Velocity, $v = s/\Delta t$, Lenticity, $u = t/\Delta s$ so $u = 1/v$ and $v = 1/u$ except if u or v are zero
  - Zero velocity: no motion but time changes because time is independent
  - Zero lenticity: no motion but length changes because length is independent
  - Circular motion in length and duration space

stance, s; time, t; radius r; period radius q; circumference $S = 2\pi r$ = wavelength λ; period $T = 2\pi q$ = wave duration μ; wave velocity, v; wave lenticity, u; arc length, s; arc time, t

- Circle in length space
  - length angle θ, arc length s, radius r
  - angle in space: $\theta \equiv s/r$; $r = s/\theta$; $s = r\theta$; $1/\theta = r/s$; $1/r = \theta/s$
  - angular time rate: $\omega \equiv \theta/t$; $t = \theta/\omega$; $\theta = \omega t$; $1/\omega = t/\theta$; $1/t = \omega/\theta$
- Circle in duration space
  - duration angle φ, arc time t, period radius q
  - angle in time, $\varphi \equiv t/q$; $q = t/\varphi$; $t = q\varphi$; $1/\varphi = q/t$; $1/q = \varphi/t$
  - angular stance rate: $\kappa \equiv \varphi/s$; $s = \varphi/\kappa$; $\varphi = \kappa s$; $1/\kappa = s/\varphi$; $1/s = \kappa/\varphi$

Angular time rates

- Independent variable is time, dependent variable is length
- Angular velocity: time rate of rotation, $\omega \equiv \theta/t$; $t = \theta/\omega$; $\theta = \omega t$

  Wave (phase) velocity: wavelength per unit time, $v = s/t = S/T = r/q = \omega r$
  - frequency, $f \equiv 1/T = v/S = v/\lambda = v/s = vh = h/u$
  - wavelength, $\lambda = v/f = S$
  - wavenumber, $\kappa = 2\pi/\lambda$
- Wave velocity normalized
  - revolutions: If $S = 1$, then $v = 1/T = f$
  - space radians: If $r = 1$, then $s = \theta = \varphi$ and $v = \theta/t = \varphi/t = s/t = \omega = 1/q = 2\pi/T = 2\pi f$
    - $\omega = 2\pi f = 2\pi/T = \theta/t = \varphi/t$; $q = T/2\pi$; $f = \omega/2\pi$

Angular stance rates

- Independent variable is stance, dependent variable is time
- Wavenumber (angular lenticity): space rate of rotation, $\kappa \equiv \varphi/s$; $s = \varphi/\kappa$; $\varphi = \kappa s$
- Wave (phase) lenticity: wave duration per unit length, $u = t/s = T/S = q/r = \kappa q$
  - periodicity, $h \equiv 1/S = u/T = u/\mu = s/v = uf = f/v$
  - wave duration, $\mu = u/h = T$
  - wave duration number, $\ell = 2\pi/\mu$
- Wave lenticity normalized
  - revolutions: If $T = 1$, then $u = 1/S = h$
  - time radians: If $q = 1$, then $t = \varphi = \theta$ and $u = \varphi/s = \theta/s = t/s = \kappa = 1/r = 2\pi/S = 2\pi h$
    - $\kappa = 2\pi h = 2\pi/S = \theta/s = \varphi/s$; $r = S/2\pi$; $h = \kappa/2\pi$

Wave function for sinusoidal wave

length amplitude, A; duration amplitude, B; length phase, ϕ; duration phase, χ; stance, s; time, t; circumference S = wavelength λ; period T = wave duration μ; wave (phase) velocity, v; wave (phase) lenticity, u

length space with time:

$x = A \cos(\omega t + \phi)$ $a = -\omega^2 x$ in SHM

$y(x = 0, t) = A \cos(\omega t) = A \cos(2\pi f t)$

$y(x, t) = A \cos[\omega(t - x/v)] = A \cos[2\pi f\,(x/v - t)]$ = sinusoidal wave moving in the $+x$-direction

$y(x, t) = A \cos[2\pi\,(x/\lambda - t/\mathrm{T})] = A \cos(kx - \omega t)$

$\partial^2 y(x, t)/\partial x^2 = (1/v^2)\, \partial^2 y(x, t)/\partial t^2$ (length) wave equation

duration space with stance:

$\xi = B \cos(\kappa s + \chi)$ $b = -\kappa^2 \xi$ in SHM

$\eta(\xi = 0, s) = B \cos(\kappa s) = B \cos(2\pi h s)$

$\eta(\xi, s) = B \cos[\kappa(s - \xi/u)] = B \cos[2\pi h\,(\xi/u - s)]$ = sinusoidal wave moving in the $+\,\xi$-direction

$\eta(\xi, s) = B \cos[2\pi\,(\xi/\mu - s/\mathrm{S})] = B \cos(\ell\xi - \kappa s)$

$\partial^2 \eta(\xi, s)/\partial \xi^2 = (1/u^2)\, \partial^2 \eta(\xi, s)/\partial s^2$ duration wave equation

Parametric equations

$|\mathbf{w}| = t = \sqrt{(t_1^2 + t_2^2)}$ $|\mathbf{r}| = s = \sqrt{(s_1^2 + s_2^2)}$

$\mathbf{r}(\theta) = r \cos(\theta)\, \mathbf{i} + r \sin(\theta)\, \mathbf{j}$
$\mathbf{r}(t) = r \cos(\omega t)\, \mathbf{i} + r \sin(\omega t)\, \mathbf{j}$; $v(t) = \omega r$; $\mathbf{a}(t) = -\omega^2 \mathbf{x}(t)$
$\mathbf{r}(x) = r \cos(x/r)\, \mathbf{i} + r \sin(x/r)\, \mathbf{j}$

$\mathbf{w}(\varphi) = q \cos(\varphi)\, \mathbf{i} + q \sin(\varphi)\, \mathbf{j}$
$\mathbf{w}(s) = q \cos(\kappa s)\, \mathbf{i} + q \sin(\kappa s)\, \mathbf{j}$; $u(s) = \kappa q$; $\mathbf{b}(s) = -\kappa^2 \mathbf{t}(s)$
$\mathbf{w}(\xi) = q \cos(\xi/q)\, \mathbf{i} + q \sin(\xi/q)\, \mathbf{j}$