Circular/harmonic motion | revised 2023-04-14

Angular velocity and angular lenticity (wavenumber)

• Velocity, *v* = *s*/Δ*t*, lenticity, *u* = *t*/Δ*s* so *u* = 1/*v* and *v* = 1/*u* except if *u* or *v* are zero

* Zero velocity: no motion but time increases because time is independent
* Zero lenticity: no motion but distance increases because distance is independent
* Circular motion in length space and duration space:



circumference *S* = 2π*r* = wavelength *λ*; period *T* = 2π*q* = 1/*f* = wave duration *μ*; arc length, *s*; arc duration, *t*; radius *r*; period radius *q*; time rate of rotation *ω*; distance rate of rotation *ψ*; angular wavenumber or repetency *k*; phase|wave velocity, *vp* = *λ*/*T* = *λf* = *ω*/*k*; phase|wave lenticity, *up* = *T*/*λ* = *k*/*ω*

Circular motion in length space with time (3+1)

* length space angle *θ*, arc length *s*, radius *r*
* length space angle *θ* ≡ *s*/*r*

Time rates of rotation

* Independent variable is time, *t*; dependent variable is arc length, *s*
* Angular velocity: time rate of rotation, *ω* ≡ *θ*/*t*
* Wave (phase) velocity: wavelength per unit time, *v = s*/*t* = *S*/*T* = *r*/*q* = *ωr*
* frequency, *f* ≡ 1/*T* = *v*/*S* = *v*/*λ* = *v*/*s*
* wavelength, *λ* = *v*/*f* = *S*
* angular wavenumber or repetency, *k* = 2π/*λ*
* Wave velocity normalized
* revolutions: If *S* = 1, then *v* = 1/*T* = *f*
* length radians: If *r* = 1, then *s* = *θ* and *v* = *θ*/*t* = *ϕ*/*t* = *s*/*t* = *ω* = 2π/*T* = 2π*f*
	+ *ω* = 2π*f* = 2π/*T* = *θ*/*t*

Circular motion in duration space with distance (1+3)

* duration space angle *ϕ*, arc duration *t*, period radius *q*
* duration space angle *ϕ* ≡ *t*/*q*

Distance rates of rotation

* Independent variable is distance, *s*; dependent variable is (dis)time, *t*
* Wavenumber (angular lenticity): distance rate of rotation: *ψ* ≡ *ϕ*/*s*
* Wave (phase) lenticity: wave duration per unit distance, *u = t*/*s* = *T*/*S* = *q*/*r* = *ψq* periodicity, *h* ≡ 1/*S* = *u*/*T* = *u*/*μ* = *s*/*v*
* wave duration, *μ* = *u*/*h* = *T*
* wave duration number, *ℓ* = 2π/*μ*
* Wave lenticity normalized
* revolutions: If *T* = 1, then *u* = 1/*S* = *h*
* duration radians: If *q* = 1, then *t* = *ϕ* = *θ* and *u* = *ϕ*/*s* = *θ*/*s* = *t*/*s* = *ψ* = 2π/*S* = 2π*h*
	+ *ψ* = 2π*h* = 2π/*S* = *θ*/*s* = *ϕ*/*s*

Coordinates

 length space (*r*, *θ*) duration space (*q*, *ϕ*)



Wave function for longitudinal wave: cosine wave

length amplitude, *A*; duration amplitude, *B*; length phase, *θ* = *ψx*; duration phase, *ϕ* = *ωt*; distance, *s*; (dis)time, *t*; circumference *S* = wavelength *λ*; period *T* = wave duration *μ*; wave (phase) velocity, *v*; wave (phase) lenticity, *u*

length space with scalar time (3+1):

*x* = *A* cos(*ωt* + *θ*) *a* =−*ω*²*x* in SHM

*y*(*x* = 0, *t*) = *A* cos(*ωt*) = *A* cos(2π*ft*)

*y*(*x, t*) = *A* cos[*ω*(*t* – *x*/*v*)] = *A* cos[2π*f* (*x*/*v* − *t*)] = sinusoidal wave moving in the +*x* length direction

*y*(*x, t*) = *A* cos[2π (*x*/*λ* − *t*/*T*)] = *A* cos(*kx* – *ωt*)

∂²*y*(*x, t*)/∂*x*² = (1/*v*²) ∂²*y*(*x, t*)/∂*t*² length space wave equation

duration space with distance (1+3):

*z* = *B* cos(*ψs* + *ϕ*) *b* = −*ψ*²*z* in SHM

*η*(*z* = 0, *s*) = *B* cos(*ψs*) = *B* cos(2π*hs*)

*η*(*z, s*) = *B* cos[*ψ*(*s* – *z*/*u*)] = *B* cos[2π*h* (*z*/*u* − *s*)] = sinusoidal wave moving in the +*z* duration direction

*η*(*z, s*) = *B* cos[2π (*z*/*μ* − *s*/*S*)] = *B* cos(*ℓz* – *ψs*)

∂²*η*(*z, s*)/∂*z*² = (1/*u*²) ∂²*η*(*z, s*)/∂*s*² duration space wave equation

Wave function for transverse wave: sine wave

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Parametric equations

|**r**| = *r* = √(*r*1² + *r*2²) |**q**| = *q* = √(*q*1² + *q*2²)

**r**(*θ*) =*r*cos(*θ*) **i** +*r*sin(*θ*) **j**

**r**(*t*) =*r*cos(*ωt*) **i** +*r*sin(*ωt*) **j**;*v*(*t*) =*ωr*; **a**(*t*) = −*ω*²**x**(*t*)

**r**(*x*) =*r*cos(*x*/*r*) **i** +*r*sin(*x*/*r*) **j**

**q**(*ϕ*) =*q*cos(*ϕ*) **i** +*q*sin(*ϕ*) **j**

**q**(*s*) =*q*cos(*ks*) **i** +*q*sin(*ψs*) **j**;*u*(*s*) =*ψq*; **b**(*s*) = −*ψ*²**t**(*s*)

**q**(*z*) =*q*cos(*z*/*q*) **i** +*q*sin(*z*/*q*) **j**